ON THE SCIENCE OF WEIGHING AND MEASURING, AND THE STANDARDS OF WEIGHT AND MEASURE

I.

DURING the last few years public attention has been frequently drawn to the subject of our national weights and measures. The administrative and social questions of the improvement of our existing system, and of the proposed introduction into this country of the decimal metric system—first established in France, and now being generally adopted on the Continent of Europe, and indeed extending to the other quarters of the world—have formed the subjects of debate in every Session of Parliament, and are still awaiting solution. The scientific questions involved in the use of weights and measures have for a much longer period engaged the attention of many of our most eminent men of Science, several of whom have been members of the various Standards Commissions from time to time appointed by the British Government. These questions are also at the present time the objects of investigation and deliberation by the large body of scientific men from all civilised countries, who compose the International Metric Commission at Paris. It may, therefore, be useful to bring together and place before the public the several points involved in the science of weighing and measuring, and to give some account of our standards of weight and measure, as well as to describe in some detail the scientific construction of our existing imperial standard yard, and pound. No sufficient means have hitherto been adopted for making the general public acquainted with this part of the subject, although they are directly interested in it, the information hitherto published respecting it having been confined to a few papers in the "Philosophical Transactions," and to reports of the several Standards Commissions, and other Parliamentary Returns. Of these papers the most important are those on the construction of the imperial standard pound, by Prof. W. H. Miller, in "Phil. Trans.," 1856, and on the construction of the new imperial standard of length, by the Astronomer Royal, now Sir G. B. Airy, K.C.B., in 1857. In the following treatment of the subject use will be made of these papers, as well as of other authoritative works relating to weights and measures.

The science of weighing and measuring comprehends the following points :—

The scientific definition of weight and measure.

The authoritative establishment of fundamental units of weight and measure of length and the construction of their material representatives as primary standards, in relation to which all numerical amounts of weight and measure are to be expressed.

The establishment of determinate aliquot parts and multiples of the primary units of weight and measure, and of other units derived from them, such as the unit of measure of capacity, &c.; and the construction and verification of their material representatives, as secondary standards, by comparison with which the accuracy of all weights and measures in ordinary use is to be determined.

The scientific methods of using standard and other weights and measures in which special accuracy is required, as well as auxiliary scientific instruments, such as balances, thermometers, barometers, micrometers, and other comparing apparatus.

The determination of the just results of weighing and measuring with these scientific instruments, after allowing for all indirect influences affecting the accuracy of the direct results of weighing and measuring; for instance, differences arising from the physical composition of bodies, variations of temperature and consequently of the expansion or contraction of the several substances, changes of condition in the medium in which the comparisons are made, &c., including also a computation of the probable errors of the final results.

The whole subject will therefore be treated under the following general heads :—

I. Definitions of weight and measure.
II. Standards of imperial weight and measure.
III. Scale of multiples and parts of imperial standard units.
IV. The metric system.
V. Weighing and measuring instruments and their use.

I. *Definitions of Weight and Measure.*

Weight or *gravity* has been defined as the quality in physical bodies by which they tend towards the centre of the earth, in a line perpendicular to its surface; or it may be defined more generally as a property inherent in all bodies, by which they are drawn to some common point, called the centre of gravity, and with a velocity in proportion as they are more or less dense, and as the medium through which they pass is more or less rare.

In following out his discovery of the theory of universal attraction and gravitation, Sir Isaac Newton demonstrated, first, that the weights of all bodies at equal distances from the centre of the earth are directly proportional to the quantity of matter that each body contains; whence it follows that the weights of bodies have no dependence on their shapes or textures, and that all spaces are not equally full of matter. Up to the time of Newton the earth was considered to be spherical, but it was demonstrated theoretically by Newton, as well as by Huygens, that the earth must be flattened at the poles. Whence it was shown by Newton, secondly, that on different parts of the earth-surface, the weight of the same body is different, owing to the spheroidal figure of the earth, which causes the body on the surface to be nearer to the centre in going from the Equator towards the Poles; and that the increase of the weight is nearly in proportion to the versed sine of double the latitude, or, which is the same thing, to the square of the sine of the latitude. He assumed the weight at the Equator to that at the Pole to be in the proportion of 229 to 230, and consequently the whole increase of weight from the Equator to the Pole to be the 229th part of the weight at the Equator.

In accordance with the principle the discovery of which is ascribed to Archimedes, that all bodies immersed in a liquid suffer a loss of weight precisely equal to the weight of the liquid displaced, it was also demonstrated that a body immersed in any fluid specifically lighter than itself loses so much of its weight as is equal to the weight of a quantity of the fluid of the same bulk with itself. Hence a body loses more of its weight in a heavier fluid than in a lighter one, and therefore it weighs more in a lighter fluid than in a heavier one, for instance, more in air than in water.

The foregoing principles laid down by Newton are universally admitted as correct, with the exception of the numerical proportions of the weight of bodies at different parts of the earth's surface; for it is important to observe that Newton founded his calculation of the earth's ellipticity on the hypothesis of its being homogeneous, which is not the case; and hence he makes the equatorial diameter greater than the polar axis, as 230 is to 229. But from the numerous experiments since made with the pendulum at different parts of the earth, it has been found that the earth is not homogeneous, or composed of concentric strata of equal density, and that the ellipticity is not so great as Newton supposed.

The method of measuring the intensity of gravity on different parts of the terrestrial spheroid, by means of the seconds pendulum, is said to be due to Borda, as originally described in a Memoir inserted in vol. iii. of the *Base du Système Métrique*. From the results of Borda's experiments, made towards the close of the last century, Laplace computed the ellipticity of the earth to be $\frac{1}{330}$, but later experiments and computations of other men of science concur in making it nearly $\frac{1}{300}$.

In the Philosophical Transactions of the Royal Society for 1818, Capt. Kater has stated the results of his pendulum experiments in London, and determined the length of the pendulum vibrating seconds, or completing one vibration in $\frac{1}{86400}$ part of a mean solar day, when measured in a vacuum at the mean level of the sea and at a temperature of 62° Fahr. to be 39·13842 inches of the Standard yard, which was legalised in 1824 as the Parliamentary Standard of length. The latitude of his place of observation in London was 51° 31′ 4″ N. He subsequently made a slight correction in this determination, making the length of the seconds pendulum to be 39·13929 inches, as shown in Phil. Trans. 1819, and this length, or rather 39·1393 inches, was declared to be the true length in the Standards Act of 1824.

It was, however, discovered by Bessel that the correction which had ordinarily been applied, and was applied by Kater, for reducing the vibrations of a pendulum, as observed in ordinary air, to vibrations in a vacuum, ought to be greatly increased. The experiments were consequently repeated by Capt. (now Sir Edward) Sabine, with special reference to the form of pendulum usually employed in England. In Phil. Trans. 1821, Sir Edward Sabine has shown as the results of his experiments on the acceleration of the pendulum in different latitudes, that the mean diminution of the force of gravity from the pole to the equator was 0·0055138, in other words, that a weight of 100 lbs. at the equator would be less by 0·55138 lb. at the pole; whilst the resulting mean ellipticity of the earth deduced from his pendulum observations, was $\frac{1}{313 \cdot 6}$. Sir Edward Sabine has also shown as the result of his experiments on the length of the seconds pendulum in Greenwich Observatory, that its length vibrating 86,400 seconds in the 24 hours, at the temperature of 62° F., and in a vacuum, was found to be 39·13734 inches.

In his paper on the Yard, the Pendulum, and the Metre, Sir J. Herschel has observed that the true measure of the earth's attraction (independent of centrifugal force arising from its rotation), is best to be derived from an ideal seconds-pendulum supposed to vibrate at the extremity of the earth's polar axis; and that the mean length of the polar or of the equatorial pendulum must be derived from the general result of observations of the lines of oscillation of one and the same invariable pendulum at a multitude of geographical stations in all accessible latitudes in both hemispheres; but that no two combinations agree in giving the same precise length, in consequence of the local deviations of the intensity of gravity, due to the nature of the soil or crust of the earth, and the configuration of the ground immediately beneath and around the places of observation. And further, that since the pendulum cannot be observed at sea, the whole sea-covered surface of the globe is of necessity excluded from furnishing its quota of observations to the final or mean conclusion. Water being on the average not more than one-third the weight of an equal bulk of land, such as the earth surface consists of, and only $\frac{2}{11}$ of the mean density of the globe, the force of gravity at the surface of the sea is less than at the sea-level on land by the attractive force of as much material taken at twice the specific gravity of water (or at $\frac{4}{11}$ that of the globe), as would be required to raise the bottom to the surface.

With regard to the determination of the earth's ellipticity, as shown by actual measurements of the dimensions of our globe, and the relative length of the equatorial diameter and the polar axis of the earth, the most recent determination is that by Major Clarke, as stated in his "Comparison of Standards of Length," published in 1866. This memoir has been declared by Sir J. F. M. Herschel, to be the most complete and comprehensive discussion yet received on the subject of the earth's figure, and to be held as the ultimatum of what scientific calculation is as yet enabled to exhibit as to its true dimensions and form.

Major Clarke's results were computed, not from pendulum experiments, but from the combination of all the separate measurements of arcs of meridians in Peru, France, Prussia, Russia, Cape of Good Hope, India, and in the United Kingdom. They are as follows:—

	Feet.	Inches.	Metres.	Metres according to Capt. Kater's equivalent.
Length of Polar axis.	41,706,858	500,482,296	12,712,136	12,712,020
Longer equatorial axis (long. 15° 34′ E.)......	41,853,700	502,244,400	12,756,588	12,756,470
Shorter equatorial axis (long. 105° 34′ E.) ...	41,839,958	502,079,496	12,752,701	12,752.583
Length of meridian quadrant of Paris....	32,813,524	393,762,292	10,001,472	10,001,381
Length of minimum quadrant (long. 105° 34′ E.)...................	32,808,772	393,704,064	10,000,024	9,999,953

In computing these equivalents, Major Clarke takes the metre at the temperature of 32° F. from his own measurements to be equal to 1·09362311 yard at 62°, that is to say to 3·28086933 ft., or to 39·37043196 in., instead of the more generally received determination by Capt. Kater of 39·37079 in. The metric length according to both these equivalents is here given.

From the determination of the earth's dimensions, it may be easily computed, that the earth's ellipticity in the longitude of Paris, is $\frac{1}{288}$, whilst its mean ellipticity in all longitudes is $\frac{1}{298}$.

Hence also the mean length of a degree of latitude in the longitude of Paris is $\frac{32,813,524 \cdot 38}{90} = 364,591$ ft., or 69·05 miles. The mean diameter of the earth is 41,800,173 ft., or 7216$\frac{2}{3}$ miles, and its mean circumference 23,871 metres.

Thus not only each longitudinal meridian, but also the equator is slightly elliptical.

Sir H. James has shown in his preface to Major Clarke's paper that the longest meridian in 15° 34′ east longitude, nearly corresponds to the meridian in the eastern hemisphere which passes over the greatest quantity of land; and in the western hemisphere to that which passes over the greatest quantity of water, as it passes through the centre of the Pacific Ocean. The shortest meridian in 105° 34′ east longitude nearly corresponds to that which passes over the greatest quantity of land in Asia; and in the western hemisphere, and that which passes over the greatest quantity of land of North and South America.

The connection here shown to exist between the definition of weight and the measurement of the dimensions of our globe, leads naturally to the definition of the second principal head of the subject, viz. of measure.

Measure is generally understood to mean the determinations of a body with relation to a fixed standard unit, or the measure of extension; and it is in this sense that it will now be taken in discussing the "science of measuring."

The measure of extension comprehends

The measure of length, or linear extension;

The measure of surface, or square measure;

The measure of volume, or solid or cubic measure;

The measure of capacity, or the cubical quantity contained in any vessel for measuring dry goods, liquids, or aëriform fluids.

All these measures of extension are based upon one fixed standard unit of length; and as all measures of length vary according to their temperature from expansion or contraction, the length of the standard must be fixed at a normal temperature.

Strictly speaking, measure includes weight, which is the measure of the gravitation of bodies towards the centre of gravity. And measures of capacity also are almost universally derived, not from their cubical dimensions, but from the weight of pure water contained in them under determinate conditions as to temperature and atmospheric pressure.

The measure of temperature is based upon the observed rate of linear expansion by heat of a body selected for this purpose, generally mercury, taking as constant units the temperature of melting snow or ice, and of water boiling under determinate atmospheric pressure.

In defining measure, it should be added that it is also applied to the measure, or (as it is termed) *admeasurement* of the tonnage of ships, being a determination of the weight a ship is capable of carrying, with relation to its measure of cubic capacity; to value in relation to a monetary unit; to time and duration in relation to the unit of a mean solar day or a second, its 86,400th part; to velocity, by combining the measure of extension with that of time or duration; to mechanical work, the unit of which is a horse power, as it is commonly termed, or more properly the power of raising 33,000 lbs. one foot in one minute, thus combining the measures of linear extension, weight, and time; to angles, the unit being a degree or the 360th part of a circle described from the point of junction of the two straight diverging lines forming the angle; &c. &c. It is not, however, proposed here to refer further to these measures or to the scientific questions connected with them.

The measure of volume, or bulk of a body, as compared with that of another body differing in volume but equal in weight, is shown by its density, and is also expressed in terms of a fixed standard unit. The densities of bodies are in the direct ratios of their masses, or quantity of matter, and in the inverse ratios of their volume.

The density of a body is defined to be the mass contained in a unit of volume, when referred to a uniform standard. The specific density is to be distinguished from its specific gravity, which shows its weight in relation to its volume, also when referred to a uniform standard. The specific gravity of a body is defined to be the *weight* of a unit of its volume.

The specific gravity of a body is the quotient of its density when divided by the density of that substance which is considered as unity. Pure water is generally adopted as such unity. But since both these densities vary with the temperature—because the same invariable quantity of matter which the body contains is always distributed over its whole volume, and this is variable with the temperature; so that, generally speaking (with some exceptions, pure water, for instance, at certain temperatures), the body, at a higher temperature, has less density than at a lower temperarure—we must fix a certain temperature at which the body, as well as the water, must be considered. It is not necessary that this fixed temperature should be the same for the body and the water, its choice for both being quite arbitrary.

For bodies the most convenient standard temperature for expressing their density seems to be that of one of the fixed points of the thermometer; and the temperature of melting ice or snow (32° F. or 0° C.) is generally adopted. For pure water, there is a maximum of density which occurs at nearly 39° F. or 4° C., and this maximum density of pure water is generally adopted as the unit of density.

The sign △ prefixed to the symbol of any weight, with its numerical value following, denotes the ratio of the density of the weight at the temperature of melting snow to the maximum density of pure water.

The relation of the bulk or volume of a body to its weight is expressed both by its density and its specific gravity, these terms being often used indiscriminately. But the former term is more strictly applicable to solid bodies, and the latter to liquids and gases.

To ascertain the density of a body, it is requisite that its volume should be determined, as the density cannot be directly found. The actual volume may be determined—

1. Either by cubic measurement, when the form of the body admits of this measurement being actually made; but this occurs but rarely.

2. Or by ascertaining its specific gravity, from determining the difference of its weight when weighed in air and in water. This is the readiest and most accurate mode of determining both its volume and its density, but the immersion of a body in water is not always practicable, or it may be injurious to the body under experiment. H. W. CHISHOLM

(*To be continued.*)

NOTES

AT the Meeting of the Paris Academy of Sciences, M. Ferdinand de Lesseps was elected an "Academicien libre" in the place of M. de Verneuil, deceased. M. de Lesseps obtained 33 votes; M. Breguet, 24; MM. du Moncel, Jacquemin, and Sedillot 1 each. M. de Lesseps thus obtained 2 votes beyond the absolute majority required to render an election valid, and was therefore declared elected. The number voting, 60, was large.

THE forty-first Annual Meeting of the British Medical Association will be held in King's College, London, on Tuesday, Wednesday, Thursday, and Friday, August 5th, 6th, 7th, and 8th. The President-elect is Sir William Fergusson, Bart., F.R.S. The following are the six sections into which the meeting will be divided, and in each section a very large number of papers is already entered to be read:—Section A, Medicine; B, Surgery; C, Obstetric Medicine; D, Public Medicine; E, Psychology; F, Physiology. The sections will meet in rooms of the College appropriated for the purpose, and the Annual Museum of objects of interest in connection with medicine, surgery, and their allied sciences will be arranged in the Library of the College. The President's address will be delivered at 3 P.M. on August 5, and in the evening the Lord Mayor will hold a reception at the Mansion House. The following public addresses will be delivered:—On August 6, an address on Medicine, by Prof. E. A. Parkes, M.D., F.R.S.; on August 7, an address on Surgery, by Prof. John Wood, F.R.S.; and on August 8, an address on Physiology, by Prof. Burdon Sanderson, F.R.S. The President and Council of the Royal College of Surgeons hold a reception on the evening of August 6, and several excursions have been arranged to take place during the meeting. Altogether, to judge from the programme, the meeting promises to be a very successful one.

THE Royal Archæological Institute commenced its annual session at Exeter, on Tuesday, when the Mayor and Corporation held a reception at noon. The President, the Earl of Devon, thereafter delivered his inaugural address on the advantages of the study of Archæology, and in the afternoon an excursion took place to Rougemont Castle. In the evening, again, the Mayor held a reception in the Albert Museum. The Sectional Meetings commenced yesterday, and several interesting excursions have been arranged. The Sections are, Antiquities, Architecture, and History. One of the most attractive accompaniments of the Exeter meeting is the formation of a temporary Museum and Portrait Gallery.

THE French Association for the Advancement of Science commences its second session at Lyons on August 21, the concluding meeting to be held on August 28. As was the case a Bordeaux, there will be General Meetings, Meetings of Sections or Groups, Scientific Excursions, and Public Lectures. A